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ESTIMATION OF CAPILLARY PRESSURE CURVES FROM CENTRIFUGE MEASUREMENTS USING INVERSE METHODS

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Abstract

The centrifuge method is a standard technique for drainage and imbibition capillary-pressure curve estimation of porous media. Several methods have been proposed for the estimation of capillary-pressure curves from centrifuge experiments. These methods may be classified as direct or inverse according to how they solve the centrifuge saturation equation. In this work, we evaluate direct and inverse capillary-pressure curve estimation methods of comparable flexibility on synthetic generated datasets. We present a theoretical justification for the applicability of parameterized inverse estimation methods, derive the analytical solution of the saturation equation of a known and frequently used capillary-pressure parameterization and discuss potential benefits of inverse methods with respect to model simplicity, ease of data usage and uncertainty analysis.

Keywords: Petrophysics. Reservoir Characterization. Capillary Pressure.

1. Introduction

The centrifuge method is a standard technique for drainage and imbibition capillary-pressure curve estimation of porous media. The method consists of imposing a capillary-

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pressure profile on a rotating rock sample through centrifugal acceleration imposition in increasing rotation speed steps and recording the average sample saturation at each step.

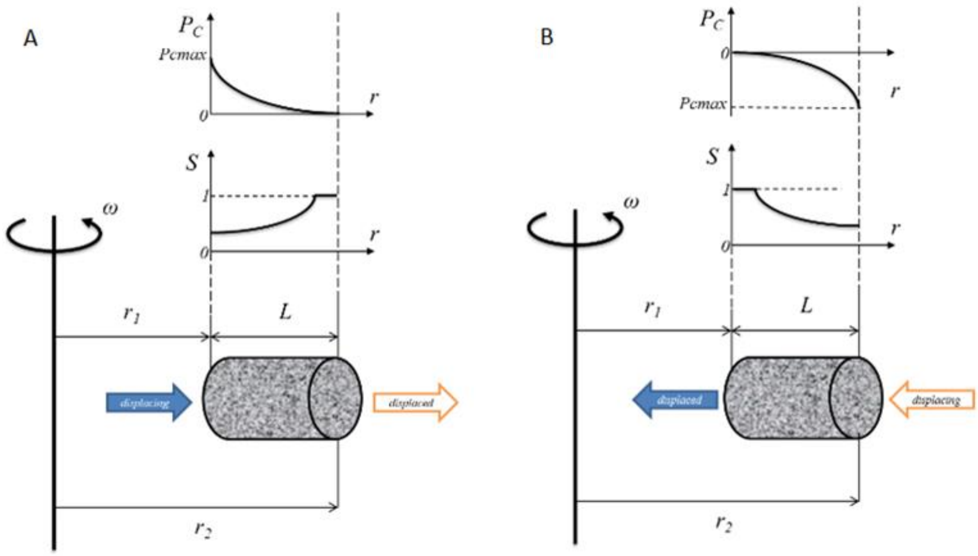

Figure 1. Drainage (A) and Imbibition (B) Centrifuge Experiment Geometries

The fundamental equations that describe the method were developed by Hassler and Brunner (HASSLER; BRUNNER, 1945):

$$P_c = \frac{1}{2}\omega^2 \Delta\rho (r^2 - r_2^2) \tag{1}$$

$$B = 1 - \left(\frac{r_1}{r_2}\right)^2 \tag{2}$$

$$S_w = \frac{V_w}{V_p} = \frac{V_w}{V_w + V_o} \tag{3}$$

$$\overline{S_w}(P_c) = \frac{(1+\sqrt{1-B})}{2} \int_0^1 \frac{S_w(xP_c)}{\sqrt{1-Bx}} dx \tag{4}$$

where ω is the centrifuge rotation speed, $\Delta\rho$ is the difference in density between the displacing and displaced fluids, L is the rock sample length, r is the radius to the centrifuge rotation center, r_1 and r_2 are the radial distances to the sample extremities, V_p, V_w and V_o are the sample pore, water and oil volumes, $S_w(P_c)$ is the water saturation at a given point in the sample subjected to a P_c capillary pressure, and $\overline{S_w}$ is the sample average water saturation.

Several methods have been proposed for the estimation of capillary-pressure curves from centrifuge experiments. These methods may be classified as direct or inverse according to how they solve the centrifuge saturation equation. Direct methods use several proposed differential and integral approximations of the saturation equation to directly estimate capillary-pressure curves from centrifuge experiment measurements at discrete capillary-pressure steps (HASSLER; BRUNNER, 1945)(SKUSE; FLROOZABADL; RAMEY JR., 1992)(FORBES, 1994). Due to the sensitivity of numerical differentiation approximations to measurement noise, direct methods are commonly preceded by a smoothing step using parameterized non-linear regression of experimental average saturation measurements. Inverse methods parameterize capillary-pressure curves and solve the saturation equation using non-linear regression (BENTSEN, 1977) or linear regression with spline basis functions (NORDTVEDT; KOLLTVELT, 1991).

An extensive survey of available capillary-pressure curve estimation methods was conducted by Forbes (FORBES, 1997), reporting that the interpretation process is a main source of inaccuracy in drainage capillary-pressure curve estimation. Through the evaluation of capillary-pressure curve estimation errors on synthetic generated datasets, Forbes suggests the inverse spline linear regression method (NORDTVEDT; KOLLTVELT, 1991) and the direct non pre-smoothed second Forbes solution method (FORBES, 1994) as methods that insure reasonable accuracy for general centrifuge geometries and capillary-pressure curve formats. These methods are very flexible, commonly using parameterizations or multiple step evaluations with high number of degrees of freedom.

2. Saturation Equation Parametric Solutions

A common assumption in reservoir characterization studies is that of statistical sample exchangeability (S.MIGON; GAMERMAN; LOUZADA, 2015), that is that the petrophysical property to be characterized does not depend on sample sequence or labeling. By De Finetti’s representation theorem and its real valued extensions (BERNARDO; SMITH, 2008), such an assumption is equivalent to that of a parametric population distribution in which samples are conditionally independent given population parameters, thus theoretically justifying parametric modeling under the sample exchangeability assumption.

Several parametric formulations have been proposed for capillary-pressure curves (FORBES, 1997). For the evaluations considered in this work, the following slight modification of the homographic parameterization proposed by (GLOTIN; GENET; KLEIN, 1990) is assumed.

$$S_w(P_c) = min\left(1, \frac{1 + \alpha S_{wi}(P_c - P_d)}{1 + \alpha(P_c - P_d)}\right) \tag{5}$$

where α, S_{wi} and P_d are parameters representing shape, irreducible water saturation and initial displacement pressure, respectively.

The analytic solution of the saturation equation for this parameterization was derived and is as follows.

$$\overline{S_w}(P_c) = \frac{(1 + \sqrt{1 - B})}{2}\left(\int_0^{x_d} \frac{1}{\sqrt{1 - Bx}} dx + \int_{x_d}^{1} \frac{S_w(xP_c)}{\sqrt{1 - Bx}} dx\right) \tag{6}$$

$$x_d = \frac{P_d}{P_c} \tag{7}$$

$$\int_0^{x_d} \frac{1}{\sqrt{1 - Bx}} dx = \frac{2}{B}\left(1 - \sqrt{1 - Bx_d}\right) \tag{8}$$

$$\begin{aligned}\int_{x_d}^{1} \frac{S_w(xP_c)}{\sqrt{1 - Bx}} dx = \frac{2(1 - S_{wi})}{\sqrt{\alpha P_c(\alpha P_c + B(1 - \alpha P_d))}}\Bigg[\operatorname{atanh}\left(\sqrt{\frac{\alpha P_c(1 - Bx_d)}{\alpha P_c + B(1 - \alpha P_d)}}\right) \\ - \operatorname{atanh}\left(\sqrt{\frac{\alpha P_c(1 - B)}{\alpha P_c + B(1 - \alpha P_d)}}\right)\Bigg] + \frac{2S_{wi}}{B}\left(\sqrt{1 - Bx_d} - \sqrt{1 - B}\right)\end{aligned} \tag{9}$$

Given this known analytic solution of the saturation equation, an error evaluation of the approximate solutions proposed by Hassler-Brunner (HASSLER; BRUNNER, 1945) and Forbes (FORBES, 1994), depicted in Figure 2, was executed for the estimation of capillary-pressure curve saturations, considering a uniform discretization with 100 capillary-pressure steps and known parameters $\alpha = 0.7$, $S_{wi} = 0.25$ and $P_d = 4$ psi. Similarly, errors introduced by numerical integration of the saturation equation, considering a capillary-pressure curve with the same parameters, were estimated using trapezoidal integration on a uniform 100 capillary-pressure step grid, and using adaptive gaussian-quadrature (BLOOMFIELD, 2014).

For the evaluated curves, both numerical integration methods presented smaller average water saturation errors than the water saturation errors obtained with the direct Forbes and Hassler-Brunner methods. Thus, numerical integration methods, as proposed by (BENTSEN, 1977), can be a good alternative for the inversion of the saturation equation, avoiding approximations introduced by direct methods.

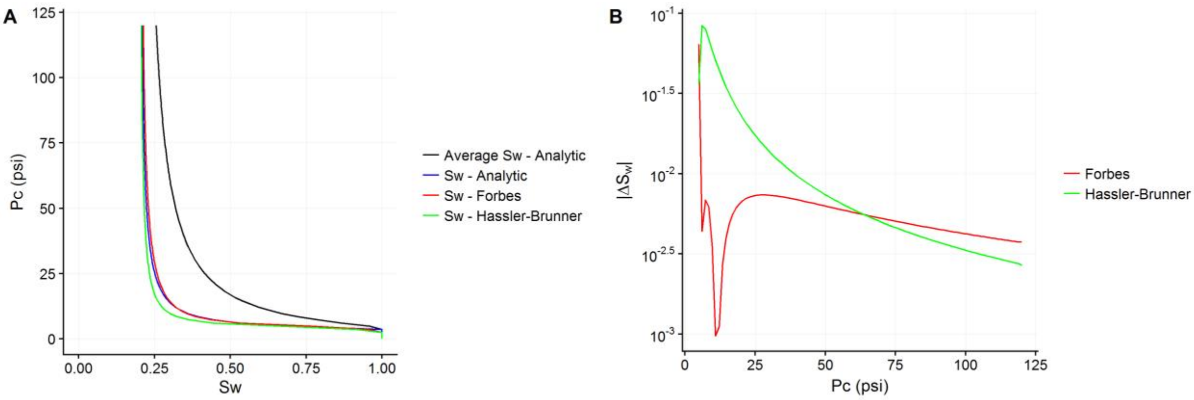

Figure 2. Capillary-Pressure Curves (A) and error evaluation (B) of direct Hassler-Brunner and Forbes methods

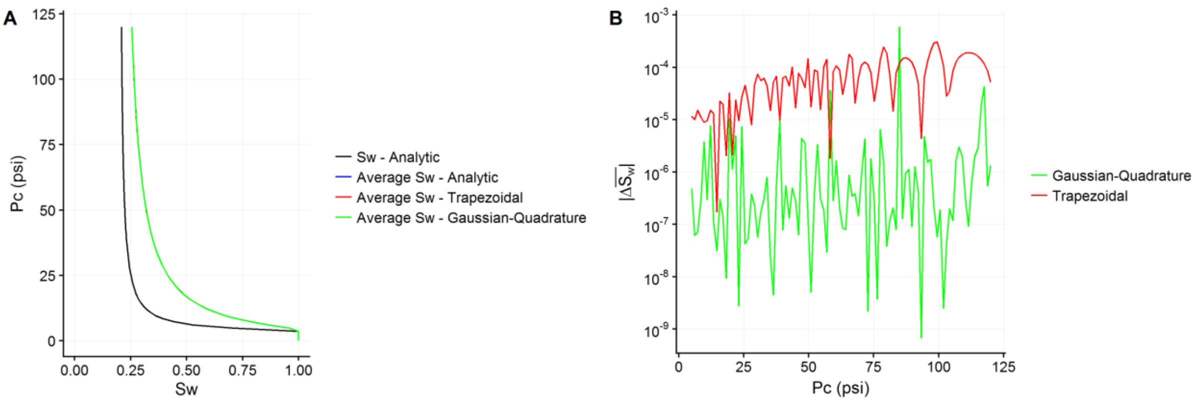

Figure 3. Capillary-Pressure Curves (A) and error evaluation (B) of trapezoidal and gaussian-quadrature numerical integration of the saturation equation

3. Capillary-Pressure Curve Estimation

A synthetic dataset was simulated using a Glotin capillary-pressure model, with parameters $\alpha = 0.3$, $S_{wi} = 0.2$ and $P_d = 1.5$ psi, and assuming measurements with additive gaussian noise, as detailed in Table 1. The standard deviation of the additive gaussian noise assumed for each rotation speed was of 2 per cent of each reference rotation speed step.

Standard deviation for measurements of oil volume production at each rotation step was fixed and assumed to be 0.2 cm³

Table 1. Simulated Dataset Parameters

Model Parameter	Value	Additive Noise Standard Deviation
V_p	10.0 cm³	0.5 cm³
L	6.00 cm	0.05 cm
r_e	11.80 cm	0.05 cm
ρ_o	0.755 g/ cm³	0.002 g/ cm³
ρ_w	1.022 g/ cm³	0.002 g/ cm³

The synthetic generated measurements were fit using the direct Forbes method and inverse non-linear regression. Prior to the application of the Forbes method, smoothing of the average water saturation measurements was performed using a non-linear regression fit, assuming a parameterization according to equation 5. The inverse non-linear regression method was performed using Maximum Likelihood Estimation (MLE), assuming a parameterization according to equations 5 and 6.

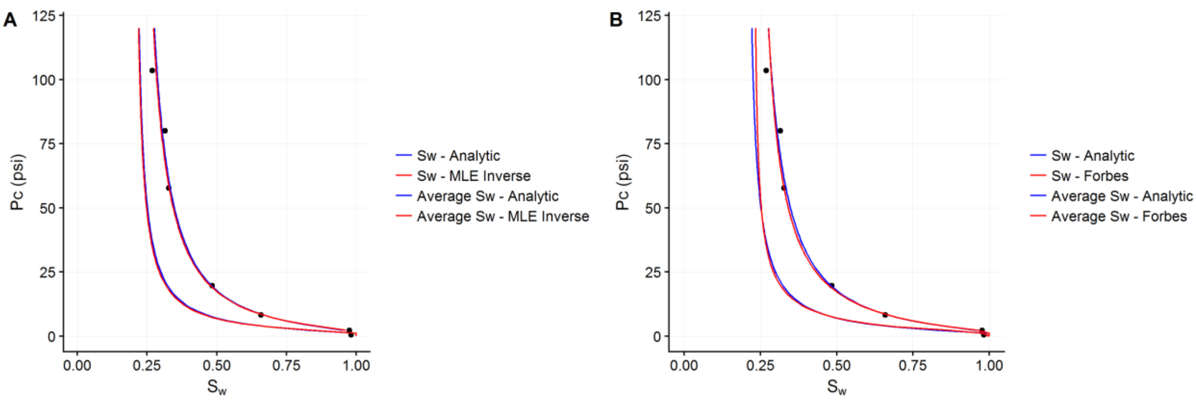

Figure 4. Inverse Maximum Likelihood Estimation (A) and direct pre-smoothed Forbes estimation (B) of capillary-pressure curves on a synthetic generated dataset, depicted as black dots, assuming additive gaussian noise

The estimated capillary-pressure curves were compared to the analytical capillary-pressure curves considering the known simulated parameters. Water saturation estimated error curves are presented on Figure 5. For the evaluated dataset, the observed inverse non-linear regression Maximum Likelihood water saturation estimate errors were smaller than the estimated errors obtained using the direct pre-smoothed Forbes method.

As equations 5 and 6 are function parameterizations with the same number of parameters and of similar flexibility, non-linear regression fits of average water saturation curves using these equations are bound to fit experimental data with comparable regression errors. Thus, inverse methods are likely to result in comparable to smaller estimated water saturation errors when compared to pre-smoothed direct methods.

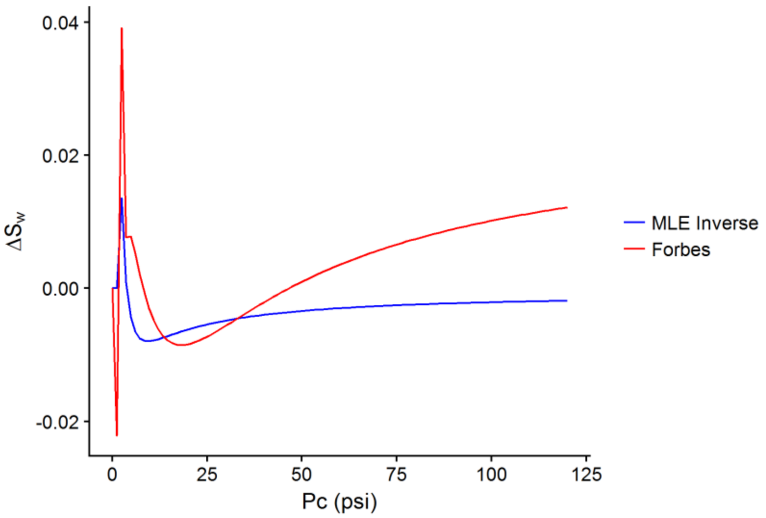

Figure 5. Water saturation error from capillary-pressure curve estimation using the inverse non-linear regression Maximum Likelihood method and the direct pre-smoothed Forbes method

4. Uncertainty Estimation

A Laplace multivariate normal approximation of the posterior distribution of the capillary-pressure curve parameters was estimated using the curvature of the negative log-likelihood function of the synthetic generated dataset at the Maximum Likelihood estimated parameters, estimated using the hessian of the negative log-likelihood function (P. MURPHY, 2012).

$$\hat{p}(\theta|\mathcal{D}) \approx \mathcal{N}(\theta|\theta_{\mathrm{mle}}, H^{-1}) \tag{10}$$

The Maximum Likelihood estimated model parameters and their respective estimated standard deviations are listed on Table 2. Samples of the parameters posterior distribution and their corresponding capillary pressure curves are displayed on Figures 6 and 7. The parameter pairs V_p and S_{wi}, and α and P_d show significant correlations, of 0.87 and 0.70 respectively.

Table 2. Maximum Likelihood Estimated Parameters

Model Parameter	Estimated Value	Estimated Standard Deviation
V_p	9.9 cm³	0.5 cm³
P_d	1.64 psi	0.47 psi
α	0.32	0.07
S_{wi}	0.20	0.05
L	5.98 cm	0.05 cm
r_e	11.77 cm	0.05 cm
ρ_o	0.754 g/ cm³	0.002 g/ cm³
ρ_w	1.018 g/ cm³	0.002 g/ cm³

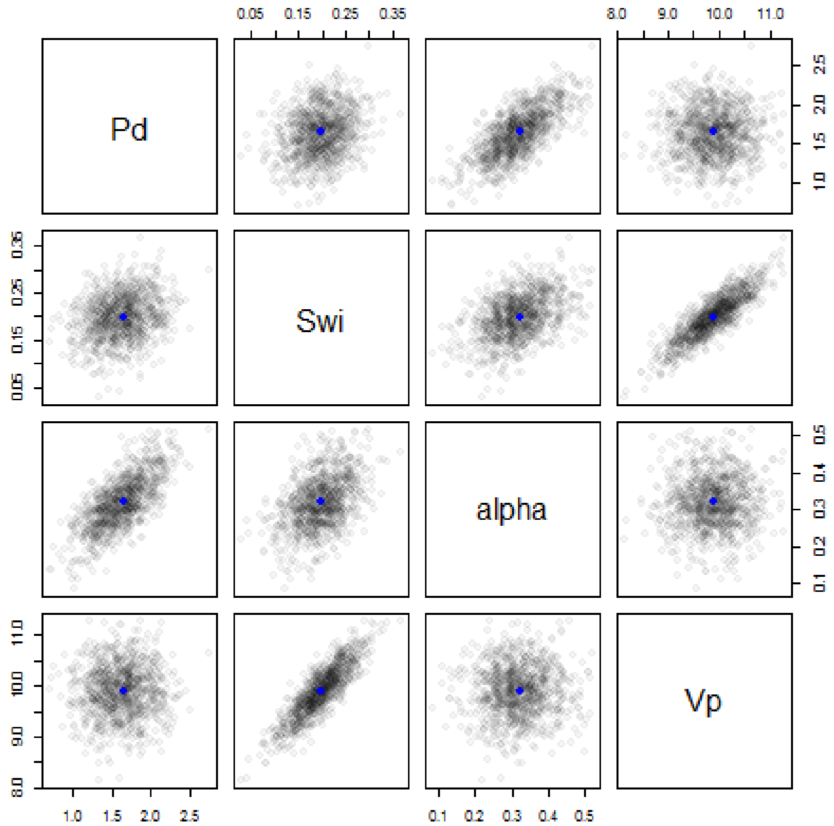

Figure 6. Maximum Likelihood multivariate normal approximation of the posterior distribution. The blue dots correspond to the true parameters used to generate the dataset

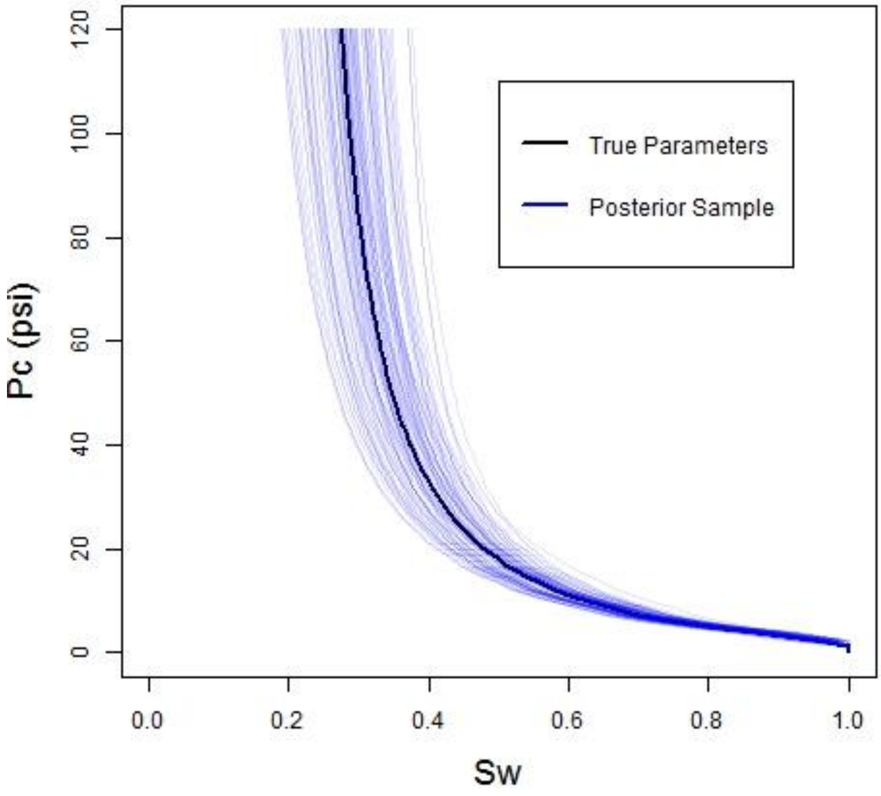

Figure 7. Capillary-pressure curves corresponding to samples of the approximate posterior distribution of the Maximum Likelihood estimated parameters

4. Conclusions

Inverse parametric estimation methods use non-linear and linear regression of analytical or numerically estimated solutions of the saturation equation to estimate capillary-pressure curves from centrifuge experiment measurements. They have the benefit of not requiring approximate solutions of the saturation equation and providing continuous capillary-pressure parametric curves, thus not requiring user interpolation of discrete capillary-pressure curves, as the ones obtained by direct methods do. Uncertainty of inverse non-linear regression parameter estimation methods can be evaluated using standard statistical procedures. For the evaluated synthetic datasets, the estimated saturation error of the inverse methods capillary-pressure curves were of lower magnitudes, and thus of comparable to slightly better performance, when compared to direct methods of equivalent flexibility.

5. Acknowledgements

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